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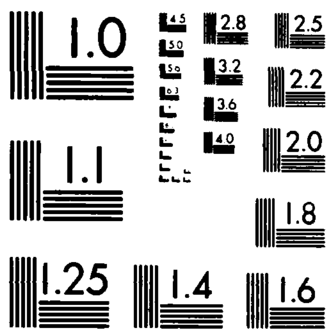

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VARIABILITY ORDERINGS RELATED TO [illegible]
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VARIABILITY ORDERINGS RELATED TO COVERAGE PROBLEMS ON THE CIRCLE

BY

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VARIABILITY ORDERINGS RELATED TO COVERAGE
PROBLEMS ON THE CIRCLE

By

Fred Huffer

1. Introduction.

Suppose that n arcs with random lengths having distributions $F_1, F_2, \ldots, F_n$ are placed uniformly and independently on a circle. Without loss of generality we assume the circle has a circumference equal to one. Questions which arise in this setting have been considered by a number of authors: Siegel (1978), Siegel and Holst (1982), Jewell and Romano (1982), Yadin and Zacks (1982), and Huffer (1982). These authors worked only with the case $F_1 = F_2 = \cdots = F_n$ of identically distributed arc lengths. The emphasis in their papers is on obtaining exact results. For example, Siegel and Holst (1982) give an exact expression for the probability that the circle is completely covered by the random arcs. However, this expression is too complicated to allow one to easily see qualitative aspects of the dependence of the coverage probability on the distribution of the arc lengths.

This article presents inequalities which tell how certain distributions and probabilities change as the variability of the distributions $F_1, F_2, \ldots, F_n$ is increased. A variability ordering is shown to hold for a fairly broad class of random variables. From this general result we obtain as immediate corollaries a number of inequalities concerning

probabilities of interest in coverage problems. In particular we are able to verify a conjecture made by Siegel (1978) concerning the probability of complete coverage.

2. Variability Orderings.

In this section we define the variability orderings and list some useful facts about them. For proofs and further details see Ross (1983) or Strassen (1965). Ross gives an elementary treatment with many examples. The treatment by Strassen is rather abstract.

Let X and Y be random variables with distributions F and G respectively. For simplicity we assume that X and Y take values in the bounded interval $[\alpha,\beta]$. This assumption suffices to handle the applications which follow.

Throughout the paper we shall use the words "increasing" and "decreasing" instead of the clumsier "nondecreasing" and "nonincreasing".

Definition: If $Eh(X) \leq Eh(Y)$ for all increasing convex functions h, then we say that $F <\uparrow G$ or equivalently $X <\uparrow Y$.

Definition: If $Eh(X) \leq Eh(Y)$ for all decreasing convex functions h, then we say that $F <\downarrow G$ or equivalently $X <\downarrow Y$.

Definition: If $Eh(X) \leq Eh(Y)$ for all convex functions h, then we say that $F <: G$ or equivalently $X <: Y$.

These orderings possess the usual properties. For example $F <: G$ and $G <: H$ imply $F <: H$. Also $F <: G$ and $G <: F$ imply $F = G$.

Note that $X <: Y$ implies $EX = EY$ and $\text{Var } X \leq \text{Var } Y$. Note also the relationship $X <\uparrow Y$ if and only if $-X <\downarrow -Y$.

We shall need the following properties:

(2.1) If $X <\uparrow Y$ and $EX = EY$, then $X <: Y$.

(2.2) $X <\uparrow Y$ if and only if $E(X-s)_+ \leq E(Y-s)_+$ for all s.

(2.3) $X <\downarrow Y$ if and only if $E(s-X)_+ \leq E(s-Y)_+$ for all s.

Here we have used $(y)_+$ to denote the positive part of y,

$$(y)_+ = \begin{cases} y & \text{for } y > 0 \\ 0 & \text{for } y \leq 0 . \end{cases}$$

Rephrasing (2.2) and (2.3) in terms of the distribution functions we obtain

(2.4) $F <\uparrow G$ if and only if $\int_s^\infty (1-F(x))dx \leq \int_s^\infty (1-G(x))dx$ for all s,

and

(2.5) $F <\downarrow G$ if and only if $\int_{-\infty}^s F(x)dx \leq \int_{-\infty}^s G(x)dx$ for all s .

(2.6) If $X <\uparrow Y$, then $P\{X=\beta\} \leq P\{Y=\beta\}$.

(2.7) If $X <\downarrow Y$, then $P\{X=\alpha\} \leq P\{Y=\alpha\}$.

The properties (2.6) and (2.7) are immediate consequences of (2.2) and (2.3).

Let T be a random variable (or vector) on the same probability space as X and Y. For all u and t define $F_t(u) = P\{X \leq u | T=t\}$ and $G_t(u) = P\{Y \leq u | T=t\}$.

$$\text{(2.8)} \qquad \begin{array}{l} \text{If } F_t <\uparrow G_t \text{ for all } t, \text{ then } F <\uparrow G . \\ \text{If } F_t <\downarrow G_t \text{ for all } t, \text{ then } F <\downarrow G . \end{array}$$

3. The Main Result.

Before stating the main result we must develop some notation. Let $X_1, X_2, \ldots, X_n$ be independent random variables uniformly distributed on the circle. These random variables will be the clockwise endpoints of the random arcs. Let $L_1, L_2, \ldots, L_n$ and $L_1', L_2', \ldots, L_n'$ be random variables which are independent of $X_1, X_2, \ldots, X_n$ and take values in the interval $[0,1]$. These variables will serve as the lengths of the random arcs. The pairs $(L_1, L_1'), (L_2, L_2'), \ldots, (L_n, L_n')$ are independent but it will be convenient in the proof to allow dependence between L_i and L_i'. The cumulative distributions of L_i and L_i' will be denoted by F_i and F_i' respectively.

If x is a point on the circle and t is a real number in the interval $[0,1]$, then $x+t$ will denote the point on the circle obtained by moving the point x a distance t in the counterclockwise direction. The point $x-t$ on the circle is defined so that $(x-t)+t = x$. The arc $[x, x+t]$ is defined by $[x, x+t] = \{x+s | 0 \leq s \leq t\}$.

From now on we shall use the variables x and y to denote points on the circle.

Let $N(y)$ be the number of times that y is covered by the arcs $[X_i, X_i+L_i]$ and $N'(y)$ be the number of times that y is covered by the arcs $[X_i, X_i+L_i']$. In symbols

$$N(y) = \sum_{i=1}^{n} I\{y \varepsilon [X_i, X_i+L_i]\}$$

and

$$N'(y) = \sum_{i=1}^{n} I\{y \varepsilon [X_i, X_i+L_i']\} .$$

An integral written without limits is understood to be an integral over the entire circle. In this situation dx denotes Lebesgue measure on the circle and $\int dx = 1$.

<u>Theorem</u>: Let $g(x,j)$ be any function which is continuous in x and increasing in the integer argument j. Define the random variables

$$W = \int g(x,N(x))dx \quad \text{and} \quad W' = \int g(x,N'(x))dx .$$

(3.1) If $F_i <: F_i'$ for all i, then $W <: W'$. The implication continues to hold if $<:$ is replaced by $<\uparrow$ or $<\downarrow$.

The theorem is a result about distributions. For convenience in the proof we have defined $N(\cdot)$ and $N'(\cdot)$ on the same probability space, but this is not an essential feature.

Proof: According to (2.1) it suffices to show that

(a) $$E\int g(x,N(x))dx = E\int g(x,N'(x))dx$$

and

(b) $$\int g(x,N(x))dx <\uparrow \int g(x,N'(x))dx \ .$$

To prove (a) we interchange the order of integration and expectation and verify that $Eg(x,N(x)) = Eg(x,N'(x))$ for all x. The interchange is justified because $g(\cdot,j)$ is a bounded function for each j in the range $0 \le j \le n$. For any fixed y the indicator functions $I\{y\varepsilon[X_i,X_i+L_i]\}$ are independent and $P\{y\varepsilon[X_i,X_i+L_i]\} = EL_i$. This remains true upon replacing L_i by L_i'. But $F_i <: F_i'$ implies $EL_i = EL_i'$. Thus $N(y)$ and $N'(y)$ have the same distribution for all y so that $Eg(y,N(y)) = Eg(y,N'(y))$.

We shall prove (b) in the special case where $F_i = F_i'$ for $1 \le i \le n-1$ and $F_n <\uparrow F_n'$. Repeated applications of this special case will show that (b) holds whenever $F_i <\uparrow F_i'$ for all i. Since we are taking $F_i = F_i'$ for $1 \le i \le n-1$ we may also assume without loss of generality that $L_i = L_i'$ for $1 \le i \le n-1$.

A final reduction of the problem is obtained by using (2.8). It suffices to show that (b) holds after conditioning on the values of $X_1,X_2,\ldots,X_{n-1}$ and $L_1,L_2,\ldots,L_{n-1}$. Define $M(y) = \Sigma_{i=1}^{n-1} I\{y\varepsilon[X_i,X_i+L_i]\}$. The variables we are conditioning on are independent of X_n, L_n and L_n' so that in what follows we may regard $M(\cdot)$ as a fixed nonrandom function

on the circle. Note that $N(y) = M(y) + I\{y\varepsilon[X_n, X_n+L_n]\}$ and $N'(y) = M(y) + I\{y\varepsilon[X_n, X_n+L'_n]\}$.

Define the function

$$W(x,u) = \int g(y, M(y) + I\{y\varepsilon[x,x+u]\})dy \ .$$

What remains to be proved is that $W(X_n, L_n) <\uparrow W(X_n, L'_n)$ whenever $F_n <\uparrow F'_n$. By (2.2) it suffices to show that $E(W(X_n,L_n)-s)_+ \leq E(W(X_n,L'_n)-s)_+$ for all s whenever $F_n <\uparrow F'_n$. Using the definition of $<\uparrow$ we need only show that $E(W(X_n,u)-s)_+$ is an increasing convex function of u for all s. Here we have used the assumption that X_n is independent of L_n and L'_n.

To verify that

$$E(W(X_n,u)-s)_+ = \int (W(x,u)-s)_+ dx$$

is an increasing convex function of u, we will show that the derivative is nonnegative and increasing in u. From the definition of $W(x,u)$ we obtain

$$\frac{\partial}{\partial u} W(x,u) = g(x+u, M(x+u)+1) - g(x+u, M(x+u))$$

so that

$$\begin{aligned}
\frac{\partial}{\partial u} \int (W(x,u)-s)_+ dx &= \int \frac{\partial}{\partial u} (W(x,u)-s)_+ dx \\
&= \int I\{W(x,u) \geq s\} \frac{\partial}{\partial u} W(x,u) dx \\
&= \int I\{W(x,u) \geq s\}[g(x+u, M(x+u)+1) - g(x+u, M(x+u))]dx \\
&= \int I\{W(x-u,u) \geq s\}[g(x, M(x)+1) - g(x, M(x))]dx \ .
\end{aligned}$$

This last integral is nonnegative since $g(x,j)$ increases with j. $W(x-u,u)$ is increasing in u so that $I\{W(x-u,u) \geq s\}$ increases in u. Therefore the integral is increasing in u as desired.

A little fuss is needed to justify the preceding formal manipulation. It is easily seen that if $x+u$ is not one of the discontinuities of $M(\cdot)$, then

$$\lim_{\delta \downarrow 0} \frac{1}{\delta}(W(x,u+\delta)-W(x,u)) = g(x+u,M(x+u)+1)-g(x+u,M(x+u)) .$$

We also have

$$\frac{1}{\delta}|W(x,u+\delta)-W(x,u)| \leq \sup_{y} (g(y,n)-g(y,0))$$

for all x and u. Thus, dividing by δ in the inequality

$$\begin{aligned} &\int dx\ I\{W(x,u) \geq s\}[W(x,u+\delta)-W(x,u)] \\ &\quad \leq \int dx(W(x,u+\delta)-s)_{+} - \int dx(W(x,u)-s)_{+} \\ &\quad \leq \int dx\ I\{W(x,u+\delta) \geq s\}[W(x,u+\delta)-W(x,u)] , \end{aligned}$$

letting δ go to zero through positive values and applying the bounded convergence theorem, we obtain the desired expression for $\frac{\partial}{\partial u} \int dx(W(x,u)-s)_{+}$. This completes the proof of (b).

The argument involved in showing that (3.1) holds with $<:$ replaced by $<\downarrow$ is almost identical to the argument for (b) given above. However,

now we must verify that $W(X_n,L_n) <\downarrow W(X_n,L_n')$ whenever $F_n <\downarrow F_n'$. By (2.3) it suffices to prove that

$$E(s-W(X_n,L_n))_+ \le E(s-W(X_n,L_n'))_+$$

for all s whenever $F_n <\downarrow F_n'$. Using the definition of $<\downarrow$ we need only show that $E(s-W(X_n,u))_+$ is a decreasing convex function of u for all s.

$$E(s-W(X_n,u))_+ = \int (s-W(x,u))_+ dx$$

and

$$\begin{aligned}
\frac{\partial}{\partial u} \int (s-W(x,u))_+ dx &= \int \frac{\partial}{\partial u} (s-W(x,u))_+ dx \\
&= -\int I\{W(x,u) \le s\} \frac{\partial}{\partial u} W(x,u) dx \\
&= -\int I\{W(x,u) \le s\}[g(x+u,M(x+u)+1)-g(x+u,M(x+u))]dx \\
&= -\int I\{W(x-u,u) \le s\}[g(x,M(x)+1)-g(x,M(x))]dx
\end{aligned}$$

which is easily seen to be nonpositive and increasing in u. This finishes the proof.

It is necessary in some applications to use functions $g(x,j)$ which are only piecewise continuous in x. This requires the following slight extension of the theorem.

Extension 1: Let $g(x,j)$ be any bounded function which is measurable in x and increasing in the integer argument j. Then (3.1) holds with W and W' defined as in the theorem.

Proof: Define

$$g_\varepsilon(x,j) = \frac{1}{\varepsilon}\int_0^\varepsilon g(x+s,j)ds \ .$$

$g_\varepsilon(x,j)$ is continuous in x so that with $W_\varepsilon = \int g_\varepsilon(x,N(x))dx$ and $W'_\varepsilon = \int g_\varepsilon(x,N'(x))dx$ we have $W_\varepsilon <: W'_\varepsilon$ whenever $F_i <: F'_i$ for all i. Choose B so that $|g(x,j)| \leq B$ for all x and j. An easy manipulation leads to

$$|W_\varepsilon - W| \leq \frac{1}{\varepsilon}\int_0^\varepsilon ds \int dx|g(x,N(x-s)) - g(x,N(x))| \ .$$

Clearly $|g(x,N(x-s))-g(x,N(x))| \leq 2BI\{N(x-s) \neq N(x)\}$. Since $N(\cdot)$ has at most $2n$ points of discontinuity $\int dxI\{N(x-s) \neq N(x)\} \leq 2ns$. Substituting these results in the earlier expression yields $|W_\varepsilon - W| \leq 2Bn\varepsilon$. Similarly $|W'_\varepsilon - W'| \leq 2Bn\varepsilon$. Thus for any convex function h we have $Eh(W_\varepsilon) \to Eh(W)$ and $Eh(W'_\varepsilon) \to Eh(W')$ as $\varepsilon \to 0$. Using the definition of $<:$ we then conclude that $W <: W'$. The argument for the orderings $<\uparrow$ and $<\downarrow$ is the same.

We shall also need a special case of another extension of the theorem.

<u>Extension 2</u>: Let λ be any finite measure on the circle and $g(x,j)$ be any function which is continuous in x and increasing in j. Define $W = \int g(x,N(x))\lambda(dx)$ and $W' = \int g(x,N'(x))\lambda(dx)$. Then (3.1) holds true.

Proof: By smoothing the measure λ we can construct a sequence $p_1, p_2, p_3, \ldots$ of nonnegative continuous functions on the circle which

satisfies $\int\lambda(dx) = \int p_k(x)dx$ for all k and $\lambda(A) = \lim_{k\to\infty} \int_A p_k(x)dx$ for all arcs A whose endpoints are not atoms of the measure λ. Define $W_k = \int g(x,N(x))p_k(x)dx$ and $W_k' = \int g(x,N'(x))p_k(x)dx$. The theorem implies that $W_k <: W_k'$ whenever $F_i <: F_i'$ for all i. With probability one, none of the discontinuities of the function $g(x,N(x))$ is an atom of λ. Thus $W_k \to W$ almost surely as $k \to \infty$. Similarly $W_k' \to W'$ almost surely as $k \to \infty$. The random variables W, W', W_k and W_k' for $1 \leq k < \infty$ are uniformly bounded so that for any convex function h we have $Eh(W_k) \to Eh(W)$ and $Eh(W_k') \to Eh(W')$ as $k \to \infty$. Using the definition of $<:$ then gives us $W <: W'$. The argument for the orderings $<\uparrow$ and $<\downarrow$ is the same.

By taking λ to be the counting measure of the set $\{x_1,x_2,\ldots,x_m\}$ we obtain the special case of greatest interest.

<u>Corollary 1</u>: Let $x_1,x_2,\ldots,x_m$ be arbitrary points on the circle and $g_1,g_2,\ldots,g_m$ be arbitrary increasing functions. Define

$$W = \sum_{i=1}^{m} g_i(N(x_i))$$

and

$$W' = \sum_{i=1}^{m} g_i(N'(x_i)).$$

Then (3.1) holds true.

4. Consequences.

This section contains applications of the previous results. In each case we obtain a variability ordering by specifying a function g and applying the results of section 3. Probability inequalities may then be derived using (2.6) or (2.7).

Example 1: Define

$$W = \sum_{i=1}^{n} L_i \quad \text{and} \quad W' = \sum_{i=1}^{n} L_i' .$$

Then (3.1) holds true.

This fact is well known and can be obtained by simple direct arguments. The result also follows immediately from the theorem after noting that with $g(x,j) = j$ for all x and j we have

$$\int g(x,N(x))dx = \sum_{i=1}^{n} L_i .$$

Example 2: Let $h(x)$ be any continuous function. Define

$$g(x,j) = I\{h(x) \leq j\} .$$

Using extension 1 we conclude that (3.1) holds with W and W' defined as in the theorem. $W = \int I\{h(x) \leq N(x)\}dx$ takes values in the interval $[0,1]$ and $P\{W=1\} = P\{h(x) \leq N(x)$ for all $x\}$. Therefore using (2.6) yields

$$(4.1) \qquad P\{N \geq h\} \leq P\{N' \geq h\} \quad \text{whenever } F_i <\uparrow F_i' \text{ for all } i .$$

Here $N \geq h$ means $N(x) \geq h(x)$ for all x. By a similar argument using (2.7) we obtain

$$(4.2) \qquad P\{N \leq h\} \leq P\{N' \leq h\} \text{ whenever } F_i <\downarrow F_i' \text{ for all } i .$$

Here $N \leq h$ means $N(x) \leq h(x)$ for all x. Since $F <: G$ implies $F <\uparrow G$ and $F <\downarrow G$, both (4.1) and (4.2) apply in the case when $F_i <: F_i'$ for all i. By taking h in (4.1) and (4.2) to be the constant function $h(x) = c$ for all x where c is an arbitrary real number, we conclude that $\inf_x N(x)$ is stochastically smaller than $\inf_x N'(x)$ whenever $F_i <\uparrow F_i'$ for all i and $\sup_x N(x)$ is stochastically larger than $\sup_x N'(x)$ whenever $F_i <\downarrow F_i'$ for all i.

<u>Example 3</u>: A special case of (4.1) is $P\{N(x) \geq 1 \text{ for all } x\} \leq P\{N'(x) \geq 1 \text{ for all } x\}$ whenever $F_i <: F_i'$ for all i. Thus the probability that the circle is completely covered increases as the variability of the distributions is increased. Let $P(n,F)$ denote the probability that the circle is completely covered by n independently and uniformly placed arcs whose lengths are independent with the distribution F. By taking $F = F_1 = F_2 = \cdots = F_n$ and $G = F_1' = F_2' = \cdots = F_n'$ in our earlier result we find that $P(n,F) \leq P(n,G)$ whenever $F <: G$. This verifies a conjecture made by Siegel (1978). $P(n,F) \leq P(n,G)$ was conjectured to hold when F was more concentrated than G in the sense that

$$G(t) \geq F(t) \quad \text{for} \quad t < \mu ,$$

and

$$G(t) \leq F(t) \quad \text{for} \quad t > \mu$$

where μ is the common mean of F and G. Using (2.1) and (2.4) or (2.5) one easily sees that this condition implies $F <: G$.

More generally, for any given set A and integer k define $Q(n,F)$ to be the probability that every point in A (except for a subset of measure zero) is covered by at least k arcs. Again we assume that n arcs with independent lengths chosen from the distribution F have been placed uniformly on the circle. By taking $g(x,j) = I\{x \in A \text{ and } j \geq k\}$ in extension 1 and then using (2.6) we *conclude that* $Q(n,F) \leq Q(n,G)$ *whenever* $F <\uparrow G$.

<u>Example 4</u>: Inequalities similar to those in the earlier examples may also be derived using corollary 1. Choose points $x_1, x_2, \ldots, x_m$ on the circle. For any given integers $k_1, k_2, \ldots, k_m$ let the functions $g_1, g_2, \ldots, g_m$ be defined by $g_i(j) = I\{j \geq k_i\}$. We conclude from corollary 1 that the random variables

$$W = \sum_{i=1}^{m} I\{N(x_i) \geq k_i\}$$

and

$$W' = \sum_{i=1}^{m} I\{N'(x_i) \geq k_i\}$$

satisfy $W <: W'$ whenever $F_i <: F_i'$ for $1 \leq i \leq n$. Similar results hold upon replacing $<:$ by $<\uparrow$ or $<\downarrow$. Using (2.6) and (2.7) we then obtain

$$P\{N(x_i) \geq k_i \text{ for all } i\} \leq P\{N'(x_i) \geq k_i \text{ for all } i\} \text{ and}$$

(4.3)

$$P\{N(x_i) < k_i \text{ for all } i\} \leq P\{N'(x_i) < k_i \text{ for all } i\}$$

whenever $F_i <: F_i'$ for $1 \leq i \leq n$.

Note: If we assume

$$F = F_1 = F_2 = \cdots = F_n \quad \text{and} \quad G = F_1' = F_2' = \cdots = F_n'$$

and let $n \to \infty$, the distributions of $(N(x_1), N(x_2), \ldots, N(x_m))$ and $(N'(x_1), N'(x_2), \ldots, N'(x_m))$ converge when suitably normalized to multivariate normal distributions with covariance matrices we shall denote by Σ and Σ' respectively. It is easily shown that $F <: G$ implies $\Sigma_{ii} = \Sigma_{jj} = \Sigma_{ii}' = \Sigma_{jj}'$ and $\Sigma_{ij} \leq \Sigma_{ij}'$ for all i and j. In the limit as $n \to \infty$ the inequalities (4.3) become special cases of Slepian's well known inequalities for the multivariate normal distribution.

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Suppose that n arcs with random lengths having distributions $F_1, F_2, \ldots F_n$ are placed uniformly and independently on a circle. This article presents inequalities which tell how certain distributions and probabilities change as the variability of the distributions $F_1', F_2', \ldots F_n'$ is increased. A distribution F is considered to be more variable than G if $\int h(x)dF(x) \geq \int h(x)dG(x)$ for all convex functions h.

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